(p,pd) Reaction

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The (p,pd) reaction with coincidence between the outgoing proton and deuteron is investigated in terms of the momentum distribution function of the two-nucleon system to be knocked out as the deuteron and the probability of finding the deuteron within the nucleus. An interaction length is defined in terms of the probability of finding the deuteron. The $Li^6(p,pd)He^4$ and $Li^7(p,pd)He^5$ reactions are considered by using single-particle harmonic-oscillator wave functions for the nucleons in the nucleus.

1. INTRODUCTION

THE momentum distribution of nucleons within the nucleus have been investigated mainly by the $(p,2p)$, (γ,p), (γ,np), and (p,d) reactions.[1–7] These reactions are very useful in investigating the momentum components for the individual nucleons in the nucleus.

The (p,pd) reaction may give information about the momentum distribution of the deuteron-cluster within the nucleus when the outgoing proton and deuteron are measured with coincidence. In this reaction the two nucleons to be knocked out as the deuteron seem to be found with appreciable probability at small separation distances within the nucleus. Then the incident proton having a wavelength less than the separation of the nucleons will be scattered elastically and the two recoiling nucleons may be observed as the deuteron. This means that in such a collision the incident proton transfers a significant part of its momentum to the two-nucleon system to be knocked out as a deuteron unit.

When the two nucleons have high relative momentum components within the nucleus, the pair interaction is expected to lead to correlations of the nucleon motion in addition to the general self-consistent field. Therefore, the (p,pd) reaction, by comparison with the known p-d elastic scattering, could yield information about the two-nucleon correlation functions and hence about the high momentum components of the nucleons in the nucleus.

2. CROSS SECTION

The cross section of the (p,pd) reaction for the momentum space element $d\mathbf{k}_pd\mathbf{k}_d$ of the final state is given by

$$\frac{d\sigma}{d\mathbf{k}_pd\mathbf{k}_d}=\frac{2\pi m}{\hbar^2k_0}|T_{fi}|^2, \tag{1}$$

where m is the proton mass. Also $\mathbf{k}_0$, $\mathbf{k}_p$, and $\mathbf{k}_d$ are the momenta of the incoming and outgoing protons and of the outgoing deuteron, respectively.

Energy conservation is given by

$$E_0=E_p+E_d+E_R+E_X+Q, \tag{2}$$

where E_0, E_p, E_d, E_R are, respectively, the kinetic energies of incoming and outgoing protons, the knocked-out deuteron and the recoil residual nucleus. E_X and Q are the excitation energy of the residual nucleus and the reaction Q value, respectively.

We assume the wave function of the target nucleus with the mass number A to be

$$\Psi=\Phi_c\psi(\mathbf{r})\psi(\mathbf{R}). \tag{3}$$

Here Φ_c denotes the wave function for the inner motion of the core nucleus with the mass number $A-2$; $\psi(\mathbf{r})$ is the wave function of the deuteron cluster to be knocked out as the deuteron; $\mathbf{r}$ indicates the relative coordinate of the two nucleons in the deuteron cluster; $\psi(\mathbf{R})$ is the wave function of the relative motion between the mass centers of the core nucleus and of the deuteron cluster; $\mathbf{R}$ refers to the coordinate between the mass centers of the core nucleus and of the deuteron cluster.

In the laboratory system the initial state is expressed as

$$\Psi_i=e^{i\mathbf{k}_0\cdot\mathbf{r}_0}D_0\Phi_c\psi(\mathbf{r})\psi(\mathbf{R}). \tag{4}$$

The final state is given by

$$\Psi_f=e^{i\mathbf{k}_p\cdot\mathbf{r}_0}D_pe^{i\mathbf{k}_d\cdot\mathbf{R}_d}D_d\psi_D(\mathbf{r})e^{i\mathbf{k}_R\cdot\mathbf{R}_c}\Phi_c. \tag{5}$$

In Eqs. (4) and (5), $\mathbf{r}_0$ is the coordinate of the incident proton referred to the mass center of the target nucleus; $\hbar\mathbf{k}_R$ is the momentum of the recoiling core nucleus; $\mathbf{R}_c$ and $\mathbf{R}_d$ are the coordinates of the mass centers of the recoiling core nucleus and the outgoing deuteron referred to the mass center of the initial target nucleus. D_0, D_p, and D_d denote, respectively, the distorting factors for the incoming and outgoing protons and the deuteron. $\psi_D(\mathbf{r})$ is the deuteron wave function given,

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[1] J. P. Garron, J. C. Jacmart, M. Riou, C. Ruhla, J. Teillac, and K. Strauch, Nucl. Phys. **37**, 126 (1962).
[2] G. Tibell, O. Sundberg, and U. Miklavzic, Phys. Letters **1**, 172 (1962).
[3] M. Q. Barton and J. H. Smith, Phys. Rev. **110**, 1113 (1958).
[4] C. Whitehead, W. R. McMurray, M. J. Aitken, N. Middlemas, and C. H. Collie, Phys. Rev. **110**, 941 (1958).
[5] A. C. Odian, P. C. Stein, A. Wattenberg, B. T. Feld, and R. Weinstein, Phys. Rev. **102**, 837 (1956).
[6] V. P. Chizhov, Zh. Eksperim. i Teor. Fiz. **38**, 809 (1960) [English transl.: Soviet Phys.—JETP **11**, 587 (1960)].
[7] P. F. Cooper and R. Wilson, Nucl. Phys. **15**, 373 (1960).

for example, by[8]

$$\psi_D=0, \quad r<r_c,$$
$$\psi_D=(N/r)[e^{-\alpha(r-r_c)}-e^{-\beta(r-r_c)}]Y_{00}\chi_{S=1,S_Z}\Omega_{t=0}, \quad r\geq r_c, \tag{6}$$

where $\alpha=0.232$ F^{-1}, $\beta=3.050$ F^{-1}, and r_c is the hard-core radius, which is about 0.4 F. Here we neglect the D-state wave function of the deuteron.

The transition matrix element is given by

$$T_{fi}=\langle\Psi_f|M|\Psi_i\rangle, \tag{7}$$

where M is the interaction operator between the incident proton and the nucleons within the deuteron cluster.

To get the momentum-conservation equation we write the interaction operator M to be the sum of the operators between the incident proton and each nucleon in the deuteron cluster:

$$M=V_{01}(\mathbf{r}_0-\mathbf{r}_1)+V_{02}(\mathbf{r}_0-\mathbf{r}_2). \tag{8}$$

Transforming the coordinates $\mathbf{R}_d$ and $\mathbf{R}_c$ to $\mathbf{R}$ and $\mathbf{Z}$ given by

$$\mathbf{R}=\mathbf{R}_c-\mathbf{R}_d,$$
$$\mathbf{Z}=(1/A)\{(A-2)\mathbf{R}_c+2\mathbf{R}_d\}, \tag{9}$$

we get the momentum-conservation equation

$$\mathbf{k}_0=\mathbf{k}_p+\mathbf{k}_d+\mathbf{k}_R, \tag{10a}$$

when we integrate over $\mathbf{Z}$ in Eq. (7) with (8). Equation (10a) is rewritten as

$$\mathbf{k}_0+\mathbf{k}=\mathbf{k}_p+\mathbf{k}_d, \tag{10b}$$

where $\hbar\mathbf{k}$ is the momentum of the deuteron cluster within the nucleus before the collision. We hold this momentum-conservation equation (10b) to be valid in the (p,pd) reaction although the interaction operator M in the transition matrix element (7) may not be approximated by that given by (8).

The transition matrix element (7) becomes

$$T_{fi}=\frac{1}{(2\pi)^{3/2}}M_{fi}\int\psi(\mathbf{r})\psi_D(\mathbf{r})e^{(i/2)(\mathbf{k}_0-\mathbf{k}_p)\cdot\mathbf{r}}d\mathbf{r}$$
$$\times\frac{1}{(2\pi)^{3/2}}\int\psi(\mathbf{R})e^{i(\mathbf{k}_0-\mathbf{k}_p-\mathbf{k}_d)\cdot\mathbf{R}}D_0D_pD_dd\mathbf{R}, \tag{11}$$

where M_{fi} is the proton-deuteron scattering matrix element connecting the initial and final states. Here we have assumed that the average proton-neutron separation in the deuteron is small compared with the nuclear dimension. Then the deuteron may be approximately treated as behaving like a single-particle unit in the nucleus. In the case of the interaction operator (8) we have

$$M_{fi}=\sum_{j=1}^{2}\int e^{-i(\mathbf{k}_0-\mathbf{k}_p)\cdot(\mathbf{r}_0-\mathbf{r}_j)}V(\mathbf{r}_0-\mathbf{r}_j)d(\mathbf{r}_0-\mathbf{r}_j). \tag{12}$$

In the semiclassical approximation, the distorting factors $D_j(j=0,p,d)$ are given by[9]

$$D_j(\mathbf{R})=\exp\left[-\frac{ik_j}{2E_j}\int V_jd\mathbf{S}_j\right], \tag{13}$$

where V_j is the optical potential (real and imaginary parts), and the integration has to be taken over the classical path of the incoming and outgoing particles, i.e., from $-\infty$ to the point of collision $\mathbf{R}$ for the incoming proton and from this point $\mathbf{R}$ to $+\infty$ for the outgoing proton and deuteron following their respective classical paths. We neglect reflection and the bending of the particles' paths caused by refraction taking the paths as straight lines in Eq. (13).

We may take the integration over r from zero to the interaction length $\bar{r}$, where $\pi\bar{r}^2$ gives approximately the average total p-p and p-n scattering cross section. This means that the collision takes place during the time when the two nucleons within the nucleus are at a close distance less than $\bar{r}$ and are strongly interacting and hence correlating. The incident proton can transfer its momentum to the tight nucleon-pair as a whole within the nucleus. Therefore the square of this term is considered to be proportional to the probability of finding the deuteron at a close distance less than $\bar{r}$. The interaction length used here is different from the value taken by Blokhintsev,[10] who has used the range of the strong interaction, i.e., $(2\sim3)\hbar/mc$ to estimate the total cross section of the elastic scattering of protons by deuterons. The value used by Blokhintsev does not depend on the momentum of the incident particle.

For elastic p-d scattering the transition matrix element is given by

$$T_{fi}'=\langle\Psi_f'|M|\Psi_i'\rangle, \tag{14}$$

where

$$\Psi_i=e^{i\mathbf{k}_0'\cdot\mathbf{r}_0}\psi_D(\mathbf{r}) \tag{15a}$$

and

$$\Psi_f=e^{i\mathbf{k}_p'\cdot\mathbf{r}_0+i\mathbf{k}_d'\cdot\mathbf{R}_d}\psi_D(\mathbf{r}). \tag{15b}$$

Here the primes indicate free elastic p-d scattering.

If we use the interaction operator given by (8) in (14) and integrate over $\mathbf{R}_d$, we have the momentum-conservation equation

$$\mathbf{k}_0'=\mathbf{k}_p'+\mathbf{k}_d'. \tag{16}$$

We hold this momentum-conservation equation to be valid in elastic p-d scattering although the interaction

[8] H. Feshbach, Phys. Rev. **107**, 1626 (1957).

[9] C. P. McCauley and G. E. Brown, Proc. Phys. Soc. (London) **71**, 893 (1958).

[10] D. I. Blokhintsev, Zh. Eksperim. i Teor. Fiz. **33**, 1295 (1957) [English transl.: Soviet Phys.—JETP **6**, 995 (1958)].

operator M in the transition matrix element (14) may not be approximated by that given by (8).

Thus we get the differential cross section for elastic p-d scattering as

$$\frac{d\sigma}{d\Omega_p'}(p-d)=\left(\frac{2m}{4\pi\hbar^2}\right)^2\frac{k_p'}{k_0'}|M_{fi}|^2 \cdot\left|\int\psi_D^2(\mathbf{r})e^{(i/2)(\mathbf{k}_0'-\mathbf{k}_p')\cdot\mathbf{r}}d\mathbf{r}\right|^2. \quad (17)$$

Since the large momentum of the incident proton is transferred to both the nucleons of the deuteron, the integration over r is limited to be from zero to the interaction length $\bar{r}$.

We replace the proton-deuteron scattering matrix element M_{fi} in (11) of the (p,pd) reaction by the elastic proton-deuteron scattering cross section. Substituting Eqs. (11) and (17) into (1) and using $|\mathbf{k}_0|=|\mathbf{k}_0'|$, we obtain the angular correlation distribution of the (p,pd) reaction to be

$$\frac{d\sigma}{d\Omega_p d\Omega_d dE_p dE_d}=\frac{m_d k_p k_d}{\hbar^2 k_p'}\frac{d\sigma}{d\Omega_p'}(p-d) \times\left|\frac{1}{(2\pi)^{3/2}}\int\psi(\mathbf{R})e^{i(\mathbf{k}_0-\mathbf{k}_p-\mathbf{k}_d)\cdot\mathbf{R}}D_0D_pD_d d\mathbf{R}P_d\right|^2 \times\delta(E_0-E_p-E_d-E_R-E_X-Q), \quad (18)$$

where m_d is the mass of the deuteron, and

$$P_d=\int\psi\psi_D e^{(i/2)(\mathbf{k}_0-\mathbf{k}_p)\cdot\mathbf{r}}d\mathbf{r}\Big/\int\psi_D^2 e^{(i/2)(\mathbf{k}_0'-\mathbf{k}_p')\cdot\mathbf{r}}d\mathbf{r} \quad (19)$$

defines the square root of the probability of finding the deuteron at a close distance within the nucleus. The momentum transfers are so large that the process is due to the very high harmonics of the pair-nucleon wave function, i.e., to those states in which the nucleons are very close together. Then we take the integration over r from zero to the interaction length $\bar{r}$.

In the derivation of Eqs. (18) and (19), we have neglected off-energy shell effects and effects (not already included in the free proton-deuteron scattering cross section) due to antisymmetrization of the wave function of the incident particle and the nuclear ones. The energy of the incident proton is large compared with the average nuclear potential. Therefore we expect the off-energy shell effects not to be serious, although we have no rigorous proof of this statement. The effects due to antisymmetrization of the wave function of the incident particle and the nuclear ones have been shown to be small in the scattering of nucleons by nuclei[11] if antisymmetrization of the wave function of the colliding two particles is taken correctly. We can expect the effects due to antisymmetrization neglected in the derivation of (18) and (19) to be small.

The angular correlation distribution (18) of the (p,pd) reaction is a function of the momentum transfers $\mathbf{k}_0-\mathbf{k}_p-\mathbf{k}_d$, $\mathbf{k}_0-\mathbf{k}_p$ and $\mathbf{k}_0'-\mathbf{k}_p'$. The probability P_d^2 of finding the deuteron within the nucleus is a function of $\mathbf{k}_0-\mathbf{k}_p$ and $\mathbf{k}_0'-\mathbf{k}_p'$. It does not, however, depend strongly on those since both the denominator and numerator of P_d contain $\mathbf{k}_0'-\mathbf{k}_p'$ and $\mathbf{k}_0-\mathbf{k}_p$, respectively. $\mathbf{k}_0'-\mathbf{k}_p'$ may be taken as $\mathbf{k}_0-\mathbf{k}_p$ when $\mathbf{k}_0-\mathbf{k}_p-\mathbf{k}_d$ is close to zero. As long as $|\mathbf{k}_0'-\mathbf{k}_p|\bar{r}\ll\pi$ and $|\mathbf{k}_0-\mathbf{k}_p|\bar{r}\ll\pi$, P_d depends on the interaction length $\bar{r}$ rather insensitively, since the integration over r is limited to be from zero to $\bar{r}$ in both the denominator and numerator. Therefore the pattern of the angular correlation distribution is mainly governed by the momentum transfer $\mathbf{k}_0-\mathbf{k}_p-\mathbf{k}_d$. When a two-nucleon system with zero angular momentum about the mass center is knocked out as a deuteron, the angular correlation distribution takes a maximum value at the angle corresponding to $\mathbf{k}_0-\mathbf{k}_p-\mathbf{k}_d=0$. If a two-nucleon system with nonzero angular momentum about the mass center is knocked out, the angular correlation distribution takes a minimum value at the angle corresponding to $\mathbf{k}_0-\mathbf{k}_p-\mathbf{k}_d=0$. It should be noted that the nuclear distortions for the incident and scattered protons and the knocked-out deuteron may slightly shift the angle giving the maximum and minimum values from the angle corresponding to $\mathbf{k}_0-\mathbf{k}_p-\mathbf{k}_d=0$.

3. NUCLEAR WAVE FUNCTION OF THE INDEPENDENT PARTICLE MODEL

The wave functions of the two nucleons within the nucleus will now be expressed in the scheme of the independent pair model. A completely antisymmetric wave function in this model may be given as

$$\Psi=\sum_{\alpha\beta}C_{\alpha\beta}\phi_\alpha(\boldsymbol{\tau}_{A-2})\phi_\beta(\mathbf{r}_1,\mathbf{r}_2), \quad (20)$$

where the symbols $C_{\alpha\beta}$ are the combinations of the fractional parentage coefficients and Wigner's coefficients which have been discussed by Elliot and Lane[12] (see Appendix B).

$$\phi_\beta(\mathbf{r}_1,\mathbf{r}_2)=f(|\mathbf{r}_1-\mathbf{r}_2|)\phi_\beta^0(\mathbf{r}_1,\mathbf{r}_2), \quad (21)$$

where ϕ_β^0 is the antisymmetric wave function describing the state β of the pair-nucleon in the independent particle model. The correlation function $f(r)$ may be taken in the form[13]

$$\begin{aligned} &f(r)=0, \quad r<r_c, \\ &f(r)=1-\exp[-\gamma\{(r/r_c)-1\}] \quad r\geq r_c, \end{aligned} \quad (22)$$

[11] G. Takeda and K. M. Watson, Phys. Rev. **97**, 1336 (1955); P. A. Benioff, Nucl. Phys. **31**, 494 (1962).

[12] T. Elliot and A. Lane, in *Handbuch der Physik*, edited by S. Flügge (Springer-Verlag, Berlin, 1957), p. 341.

[13] J. Dabrowski, Proc. Phys. Soc. (London) **71**, 658 (1958).

with $\gamma=0.75\sim2.00$. Also ϕ_α is the antisymmetric wave function of the remaining nucleons in the state α.

If β denotes the set consisting of the total angular momentum L of the pair nucleons with its projection M_L, the total spin S with its projection S_Z, and the total isospin t with its projection t_Z, then $\phi_\beta{}^0(\mathbf{r}_1,\mathbf{r}_2)$ may be expressed as

$$\phi_\beta{}^0(\mathbf{r}_1,\mathbf{r}_2)=R_1(r_1)R_2(r_2)\sum_{m_1m_2}(l_1l_2m_1m_2|LM_L)\times Y_{l_1m_1}(1)Y_{l_2m_2}(2)\chi_{SS_Z}(s_1,s_2)\Omega_{tt_Z}(t_1,t_2)\,, \quad (23)$$

where $R_{1,2}$ are the single-particle radial wave functions of the nucleons 1 and 2, and l_1, l_2 and m_1, m_2 are, respectively, their angular momentum quantum numbers and their projections. Since the deuteron wave function (6) includes the spin-triplet function, we use the L-S coupling representation for the pair nucleons.

If the radial wave functions are assumed to be harmonic-oscillator functions, then $\phi_\beta{}^0$ can be transformed to

$$\phi_\beta{}^0=\sum\langle n_1l_1,n_2l_2;L|nl,NL_0;L\rangle(lL_0mM|LM_L)\times\phi_{nlm}(\mathbf{x})\phi_{NL_0M}(\mathbf{Y})\chi_{SS_Z}\Omega_{tt_Z}\,, \quad (24)$$

where n_1 and n_2 are the radial quantum numbers, and ϕ_{nlm} and ϕ_{NL_0M} are, respectively, the harmonic-oscillator functions depending on the relative coordinate $\mathbf{x}$ and on the center-of-mass coordinate $\mathbf{Y}$ of the two-nucleon system. $\langle n_1l_1,n_2l_2;L|nl,NL_0;L\rangle$ is Talmi's transformation coefficient.[14]

The momentum $\hbar\mathbf{k}$ of Eq. (10b) is conjugate to the relative coordinate between the mass centers of the two nucleons to be knocked out and of the core nucleus. Then, when Eq. (24) is used to calculate the transition matrix element T_{fi}, the coordinate $\mathbf{Y}$ between the mass centers of the target nucleus and of the two nucleons should be replaced by $\mathbf{R}$, which is the coordinate between the mass centers of the core nucleus and of the two nucleons. Considering the reduced mass of the mass of the core nucleus and of that of the two nucleons, we change the length parameter and hence replace the coordinate $\mathbf{Y}$ by the coordinate $\mathbf{R}$ in ϕ_{NL_0M} of (24). The relative coordinate $\mathbf{x}$ between the two nucleons is replaced by $\mathbf{r}$, that is,

$$\mathbf{r}=\mathbf{x}\,. \quad (25)$$

Using the coordinates $\mathbf{R}$ and $\mathbf{r}$ for the two nucleons and the orthogonality of the wave function for the core nucleus in the state α, we obtain the angular correlation distribution for the α, β state of the target nucleus as

$$\frac{d\sigma}{d\Omega_pd\Omega_ddE_pdE_d}=\frac{m_dk_pk_d}{\hbar^2k_p'}\frac{d\sigma}{d\Omega_p'}(p-d)\sum\frac{1}{2L_0+1}\frac{1}{2l+1}\times\Bigg|C_{\alpha\beta}\sum\langle n_1l_1,n_2l_2;L|nl,NL_0;L\rangle(lL_0mM|LM_L)\times\frac{1}{(2\pi)^{3/2}}\int\phi_{NL_0M}e^{i(\mathbf{k}_0-\mathbf{k}_p-\mathbf{k}_d)\cdot\mathbf{R}}D_0D_pD_dd\mathbf{R}\times\frac{\int\psi_D\phi_{nlm}f(r)e^{(i/2)(\mathbf{k}_0-\mathbf{k}_p)\cdot\mathbf{r}}d\mathbf{r}}{\int\psi_D{}^2e^{(i/2)(\mathbf{k}_0'-\mathbf{k}_p')\cdot\mathbf{r}}d\mathbf{r}}\Bigg|^2\times\delta(E_0-E_p-E_d-E_R-E_X-Q)\,. \quad (26\text{a})$$

By using the orthogonality of the wave functions ψ_D and ϕ_{nlm}, Eq. (26a) is rewritten as

$$\frac{d\sigma}{d\Omega_pd\Omega_ddE_pdE_d}=\frac{m_dk_pk_d}{\hbar^2k_p'}\frac{d\sigma}{d\Omega_p'}(p-d)\sum\frac{1}{2L_0+1}\times\Bigg|C_{\alpha\beta}\sum\langle n_1l_1,n_2l_2;L_0|n0NL_0;L_0\rangle\times\frac{1}{(2\pi)^{3/2}}\int\phi_{NL_0M}e^{i(\mathbf{k}_0-\mathbf{k}_p-\mathbf{k}_d)\cdot\mathbf{R}}D_0D_pD_dd\mathbf{R}P_d\Bigg|^2\times\delta(E_0-E_p-E_d-E_R-E_X-Q)\,, \quad (26\text{b})$$

where

$$P_d=\frac{\int\psi_D\phi_{n00}f(r)e^{(i/2)(\mathbf{k}_0-\mathbf{k}_p)\cdot\mathbf{r}}d\mathbf{r}}{\int\psi_D{}^2e^{(i/2)(\mathbf{k}_0'-\mathbf{k}_p')\cdot\mathbf{r}}d\mathbf{r}}\,. \quad (27)$$

4. $Li^6(p,pd)He^4$ REACTION

In the present section we consider the $Li^6(p,pd)$ reaction. With a 155-MeV proton beam, Riou *et al.*[15] have measured the reaction taking into account coincidence between the outgoing proton and deuteron. They have chosen the momenta of the outgoing proton and deuteron to be $|\mathbf{k}_p|=|\mathbf{k}_d|$ and taken the scattering angle to be $\mathbf{k}_p\cdot\mathbf{k}_0=\mathbf{k}_0\cdot\mathbf{k}_d$, i.e., $\theta_p=\theta_d$, where θ_p and θ_d are the angles of the outgoing proton and deuteron referred to the direction of the incident proton.

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[14] I. Talmi, Helv. Phys. Acta **25**, 185 (1952). M. Moshinsky, Nucl. Phys. **13**, 104 (1959). A. Arima and T. Terasawa, Progr. Theoret. Phys. (Kyoto) **23**, 115 (1960).

[15] C. Ruhla, M. Riou, J. P. Garron, J. C. Jacmart, and L. Massonnet, Phys. Letters **2**, 44 (1962).

When the residual nucleus He^4 is left in its ground state in the $Li^6(p,pd)$ reaction, the p proton and p neutron are expected to be knocked out as the deuteron.

From the viewpoint of shell theory the Li^6 nucleus consists of an alpha-particle core with a p^2 configuration of nucleons about it. Assuming a pure p^2 configuration, we take the wave function for the ground state $J=0^+$, $t=0$ given by linear combination of the ${}^3S_1$, ${}^1P_1$, and ${}^3D_1$ components in the L-S coupling scheme. There is strong evidence[16] that the ${}^3S_1$ component is dominant. The experimental magnetic moment of Li^6 is very close to the theoretical value calculated by using the ${}^3S_1$ component of the wave function. The square of the beta-decay matrix element in the superallowed He^6-Li^6 transition, which is calculated by using the ${}^3S_1$ component of the ground state of Li^6 and the ${}^1S_0$ component of the ground state of He^6, can explain the experimental ft value. The ${}^3S_1$ component of the wave function in the ground state of Li^6 is written as

$$\phi(\mathbf{R},\mathbf{r})=(1/\sqrt{2})\{2S(\mathbf{r})1S(\mathbf{R}) - 2S(\mathbf{R})1S(\mathbf{r})\}\chi_{S=1,S_Z}\Omega_{t=0} \quad (28)$$

in the L-S coupling scheme. The wave function (28) is constructed from the harmonic-oscillator functions of two p nucleons such as

$$\varphi_p(\varrho)=2\left(\frac{2}{3}\right)^{1/2}\frac{1}{\pi^{1/4}}\frac{\rho}{a^{5/2}}\exp\left(-\frac{\rho^2}{2a^2}\right)Y_{1\mu}(\theta,\varphi)\,, \quad (29)$$

where a is the length parameter which may be determined by fitting the high-energy electron scattering data.[17] The explicit forms of $2S(\mathbf{r})$, $1S(\mathbf{r})$, $2S(\mathbf{R})$ and $1S(\mathbf{R})$ are

$$\begin{aligned}
2S(\mathbf{r})&=\left(\frac{2}{\pi a^6}\right)^{1/4}\frac{1}{\sqrt{6}}\left(3-\frac{r^2}{a^2}\right)\exp\left(-\frac{r^2}{4a^2}\right)Y_{00}\,,\\
1S(\mathbf{r})&=\left(\frac{2}{\pi a^6}\right)^{1/4}\exp\left(-\frac{r^2}{4a^2}\right)Y_{00}\,,\\
2S(\mathbf{r})&=\left(\frac{1}{2\pi a'^6}\right)^{1/4}\frac{4}{\sqrt{6}}\left(3-\frac{4R^2}{a'^2}\right)\exp\left(-\frac{R^2}{a'^2}\right)Y_{00}\,,\\
1S(\mathbf{R})&=\left(\frac{1}{2\pi a'^6}\right)^{1/4}4\exp\left(-\frac{R^2}{a'^2}\right)Y_{00}\,,
\end{aligned} \quad (30)$$

where

$$a'^2=\frac{A}{(A-2)}a^2. \quad (31)$$

It should be noted that the length parameters for $2S(\mathbf{R})$ and $1S(\mathbf{R})$ differ from those for $2S(\mathbf{r})$ and $1S(\mathbf{r})$ by a factor $[A/(A-2)]^{1/2}$. This is due to the fact that

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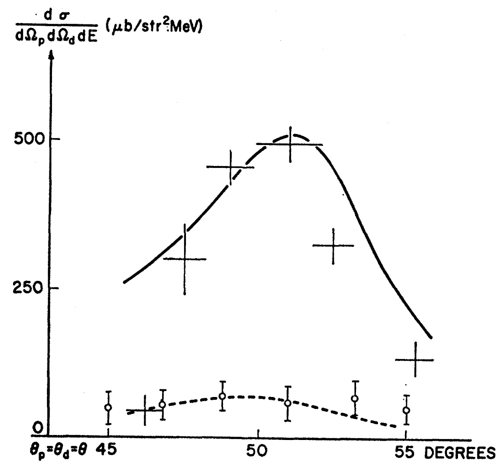

FIG. 1. The angular correlation distribution of the (p,pd) reaction. The solid and dashed lines show the calculated results for the $Li^6(p,pd)He^4$ and $Li^7(p,pd)He^5$ reactions when p nucleons are knocked out as a deuteron. The curves for the $Li^6(p,pd)$ and $Li^7(p,pd)$ reactions have been multiplied by factors 7 and 1.3, respectively.

$\mathbf{R}$ indicates the coordinate between the mass centers of the two nucleons and of the core nucleus. Equation (31) is derived by considering the reduced mass of the mass of the two nucleons and of that of the core nucleus.

Under the conditions $|\mathbf{k}_p|=|\mathbf{k}_d|$ and $\mathbf{k}_p\cdot\mathbf{k}_0=\mathbf{k}_0\cdot\mathbf{k}_p$, it follows that $\mathbf{k}_0-\mathbf{k}_p-\mathbf{k}_d=0$ at $\theta_p=51°$ in the laboratory system. The cross section for elastic p-d scattering, $(d\sigma/d\Omega_p')(p-d)$, is equal to[18] 0.89 mb at the scattering angle $\theta_p=51°$ in the laboratory system. The angular correlation distribution calculated from Eqs. (26b) and (28) is compared with the experimental data in Fig. 1. Here we have chosen the length parameter[19] $a=2.14$ F and used the square-well optical potential with the radius $R_0=3.5$ F to estimate the distorting factors D_0, D_p, and D_d. The value of R_0 is determined from the uniform charge distribution[17] of the Li^6 nucleus. The optical potential parameters are assumed to be purely imaginary, i.e.,

$$V_0=-5i\ \text{MeV}\,,\quad V_p=-4i\ \text{MeV}\,,\quad V_d=-13i\ \text{MeV}\,,$$

inside the square well with the radius R_0. Outside the radius R_0 they are put equal to zero. These potentials are expected to reproduce approximately the total scattering cross sections of protons by Li^6 at 155 and 100 MeV and of deuterons by Li^5 at 50 MeV. The radius R_0 is assumed not to change before and after the collision. The reaction Q value is taken as 1.471 MeV.[20] The parameter γ in Eq. (22) is taken as 2.0.

[18] H. Postma and R. Wilson, Phys. Rev. **121**, 1229 (1961).

[19] L. R. B. Elton, *Nuclear Sizes* (Oxford University Press, London, 1961).

[20] F. Ajzenberg-Selove and T. Lauritsen, Nucl. Phys. **11**, 1 (1958).

The absolute magnitude of the differential cross section for the angular correlation distribution calculated by using the wave function derived from the simple harmonic-oscillator functions is small compared with the experimental data obtained by Riou *et al.*[15] This was found in the preliminary note.[21] In the case of quasifree *p*-*p* scattering by the *p* proton of Li^6, it has been shown that the harmonic-oscillator wave function for the *p* proton[22,23] having the length parameter determined from the high-energy electron scattering cannot reproduce the $(p,2p)$ experimental data.[1,2] The harmonic-oscillator wave function originating in an infinite potential does not extend far enough in space and therefore gives too small an amount of low-momentum components for the *p* nucleons of Li^6. The wave function which has a large amount of low-momentum components can improve the agreement between the calculated and experimental results in the $(p,2p)$ reaction. Such wave functions for the *p* nucleons of Li^6 may be given by the cluster model. This suggests that the agreement between the calculated and the experimental results of the $Li^6(p,pd)$ reaction may be improved by using such a wave function given by the cluster model.

If the Li^6 nucleus breaks up into a deuteron and an alpha particle as a result of interaction with an incident proton, the angular correlation distribution of the $Li^6(p,p\alpha)$ reaction with coincidence between the outgoing proton and alpha particle gives complementary information about the relative motion between the mass centers of the deuteron and alpha clusters. The angular correlation distributions of both the (p,pd) and $(p,p\alpha)$ reactions are mainly governed by the relative motion between the deuteron and alpha clusters within the Li^6 nucleus.

If one *p* and one *s* nucleon are knocked out as the deuteron, the angular momentum about the mass center of the two-nucleon system is $L_0=1$ because of the symmetry of the coordinates of the two nucleons in the deuteron. This gives the minimum value for the angular correlation distribution around the angle corresponding to $\mathbf{k}_0-\mathbf{k}_p-\mathbf{k}_d=0$. If two *s* nucleons are knocked out, the angular correlation distribution shows the maximum around the angle corresponding to $\mathbf{k}_0-\mathbf{k}_p-\mathbf{k}_d=0$.

When the knocked-out deuteron consists of one *p* and one *s* nucleon or of two *s* nucleons, the residual nucleus is left in high continuum excited states, and the energy spectrum of the reaction, which is the energy sum of the outgoing proton and deuteron, does not show any peak corresponding to the individual excited state of the residual nucleus. Besides this, the large absorption effects and the very short lifetime of the *s*-state holes make the measurement for these deuterons difficult.

5. $Li^7(p,pd)He^5$ REACTION

For the *p* nucleons of Li^7 we use the harmonic-oscillator wave functions with the length parameter $a=1.99$ F.[23] The square-well optical potential with the radius $R_0=3.5$ F is taken to estimate the distorting factors D_0, D_p, and D_d. The value of R_0 is determined from the uniform charge distribution[17] of the Li^7 nucleus. The optical potential parameters within R_0 are assumed to be purely imaginary:

$$V_0=-6i\ \text{MeV},$$
$$V_p=-5i\ \text{MeV},$$
$$V_d=-13i\ \text{MeV},$$

These values are expected to reproduce approximately the total scattering cross sections of protons by Li^7 at 155 and 100 MeV and of deuterons by Li^6 at 50 MeV. The radius R_0 is assumed to be constant before and after the collision. The reaction Q value is 9.681 MeV.[20] The parameter γ is taken as 2.0.

When the knocked-out deuteron consists of *p* nucleons, the residual He^5 nucleus is left either in the ground state $P_{3/2}$ or in the very short-lived $P_{1/2}$ resonance state in the $Li^7(p,pd)$ reaction. Riou *et al.*[15] do not distinguish the contributions from the ground state and the short-lived resonance state of the residual nucleus to the differential cross section for the angular correlation distribution. To compare with the experimental data, we add the calculated cross sections for the ground and the short-lived resonance states of the residual nucleus. The results calculated under the conditions $|\mathbf{k}_p|=|\mathbf{k}_0|$ and $\mathbf{k}_p\cdot\mathbf{k}_0=\mathbf{k}_0\cdot\mathbf{k}_d$ are compared with the experimental data in Fig. 1. Here we have assumed that the incoming proton and outgoing proton and deuteron interact only elastically with the remaining nucleons of the nucleus.

When the knocked-out deuteron consists of one *p* and one *s* nucleon or two *s* nucleons of Li^7, the angular correlation distribution is expected to show the minimum or maximum around the angle corresponding to $\mathbf{k}_0-\mathbf{k}_p-\mathbf{k}_d=0$. The measurement for these deuterons would be very difficult. In addition to the effects of absorption, the residual nucleus is left in high continuum excited states and the *s*-state holes have very short lifetimes.

6. CONCLUDING REMARKS

There is some ambiguity about the optical potential used in the calculation of the $Li^6(p,pd)$ and $Li^7(p,pd)$ reactions to estimate the distorting factors (13).

The parameters of the nuclear optical potential are

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[21] Y. Sakamoto, Nuovo Cimento **28**, 206 (1963). In Eq. (8) of this note $\mathbf{k}_0+\mathbf{k}_R/\mathbf{k}_p-\mathbf{k}_d$ should read as $\mathbf{k}_0-\frac{1}{2}\mathbf{k}/\mathbf{k}_p-\frac{1}{2}\mathbf{k}_d$, and $\theta_p=77°$ should read as 73.5°. The author is indebted to Dr. L. R. B. Elton for pointing out these corrections. Also the multiplying factor for the calculated curve is 10.4 instead of 2.6. If we use the center-of-mass cross section for free *p*-*d* elastic scattering, Eq. (26b) of the present paper becomes Eq. (8) of Ref. 21.

[22] A. Johansson and Y. Sakamoto, Nucl. Phys. **42**, 625 (1963).

[23] T. Berggren and G. Jacob, Nucl. Phys. **47**, 481 (1963).

closely related[24] to the experimentally measured quantities. The imaginary parts of the optical potential are closely connected with the total scattering cross section of particles by the nucleus. Experimental data[25] are available for the total scattering cross section of protons by Li^7. From these data we can estimate the total scattering cross section of protons by Li^6. Thus we get the imaginary parts of the optical potential for the scattering of protons from Li^6 and Li^7. We know very little about the total scattering cross section of high-energy deuterons by the nucleus. Therefore, we assume the total scattering cross section of deuterons by the nucleus to be the sum of the total scattering cross section of the protons and of the neutrons having the energy equal to that of the individual nucleons of which the high-energy deuteron is composed. Then we can roughly estimate the imaginary parts of the optical potential for the scattering of high-energy deuterons from the nucleus.

The dependence of the absolute magnitude of the differential cross section for the angular correlation distribution in the (p,pd) reaction on the square-well optical potential is rather insensitive to change of the parameters as long as R_0 and V_0, V_p and V_d are chosen to reproduce the observed total scattering cross section of protons by Li^7 and those of protons by Li^6 and of deuterons by the nucleus.

We have neglected the real parts of the optical potential in the distorting factors (13). The absolute magnitude of the calculated differential cross section for the angular correlation distribution is very little sensitive to the real parts of the optical potential.

The harmonic-oscillator wave functions used in the calculation of the $Li^6(p,pd)$ and $Li^7(p,pd)$ reactions give a Gaussian-type distribution for the nuclear density. It may be desirable in the distorting factors (13) to use the Gaussian-type optical potential corresponding to the Gaussian distribution for the nuclear density. If the Gaussian-type optical potential with suitable parameters is used in the calculation, we may get better agreement between the calculated and observed angular correlation distribution.

In the present calculation we have used the $^3S_1$ component of the wave function for the ground state of Li^6 derived from harmonic-oscillator functions having the length parameter determined from the high-energy electron scattering in the L-S coupling scheme. Using this wave function we cannot reproduce the experimental data for the $Li^6(p,pd)$ reaction in the absolute magnitude of the differential cross section for the angular correlation distribution even if we put the optical potentials V_0, V_p, and V_d to be equal to zero in the distorting factors (13). The calculated results are small compared with the experimental data in the absolute magnitude for the angular correlation distribution. On the other hand, the observed absolute magnitude of the differential cross section for the angular correlation distribution of the $Li^7(p,pd)$ reaction is nearly reproduced by using harmonic-oscillator functions having the length parameter determined from the high-energy electron scattering.

The derivation of the angular correlation distribution (18) involves the approximations. These approximations are the semiclassical approximation used to take distortion effects into account, the neglect of off-energy shell effects in using the free proton-deuteron scattering cross section, and the neglect of target exchange and other related effects. The use of the square-well optical potential in the distorting factors (13) may not be proper. From the comparison between the calculated results for the $Li^6(p,pd)$ and $Li^7(p,pd)$ reactions, however, we see that, when the wave function of the relative motion between the mass centers of the core nucleus and of the two p nucleons of Li^6 gives a large amount of low-momentum components, the wave function may give better agreement with the experimental data of the $Li^6(p,pd)$ reaction. The separation energy of Li^6 into an alpha particle and a deuteron is 1.471 MeV. The mass centers of the core nucleus and of the two p nucleons are on the average rather far apart and hence the wave function of the relative motion between the mass centers of the core nucleus and of the two p nucleons gives a large amount of low-momentum components. This may suggest a cluster structure consisting of deuteron and alpha clusters for the Li^6 nucleus. These clusters behave more or less like free particles. For the deuteron-cluster it is expected that it will have a long tail just as a free deuteron does. This deuteron-cluster wave function gives a large overlap integral with the deuteron wave function, and hence gives a considerably large probability of finding the deuteron within the Li^6 nucleus. The absolute magnitude of the differential cross section for the angular correlation distribution in the $Li^7(p,pd)$ reaction is much lower than that in the $Li^6(p,pd)$ reaction. This may indicate that the Li^7 ground state is made up of alpha and triton clusters, and that there are very few deuteron clusters. Therefore it appears that the (p,pd) reaction can be a useful tool for investigating the cluster structure of nuclei.

The (p,pd) reaction takes place only at the nuclear surface instead of throughout the nuclear volume, because of the small mean free path of the deuterons in the nucleus. It could, therefore, furnish a means for examining the surface of heavier nuclei.

An examination of the (p,pd) reaction in which the residual nucleus is left in individual excited states would be very helpful for understanding the level structure of the residual nucleus.

If the momentum transfer $\mathbf{k}_0-\mathbf{k}_p$ is kept constant and θ_p is fixed, then the angular correlation distribution

[24] W. Riesenfeld and K. M. Watson, Phys. Rev. **102**, 1157 (1956). A. K. Kerman, H. McManus, and R. M. Thaler, Ann. Phys. (N. Y.) **8**, 551 (1959).

[25] A. Johansson, U. Svanberg, and O. Sundberg, Arkiv. Fysik **19**, 527 (1961).

can be measured as a function of the momentum transfer $\mathbf{k}_0-\mathbf{k}_p-\mathbf{k}_d$ by varying θ_d. In this case the probability of finding the deuteron within the nucleus remains constant, and hence more detailed information about the momentum distribution for the mass center of the two nucleons will be obtained. After investigating the momentum distribution function for the mass center of the two nucleons, we can analyze in detail the probability of finding the deuteron within the nucleus, which then gives some information about the two-nucleon correlation functions, and hence about the high-momentum components of the nucleons within the nucleus.

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APPENDIX A

The cross section of the (p,pd) reaction for the momentum space element $d\mathbf{k}_p d\mathbf{k}_d$ is

$$d\sigma/d\mathbf{k}_p d\mathbf{k}_d = A|M_{fi}|^2 f(\mathbf{k})\rho(E_B), \tag{A1}$$

with

$$\mathbf{k}_0+\mathbf{k}=\mathbf{k}_p+\mathbf{k}_d$$

and

$$E_0=E_p+E_d+E_B,$$

where

$$E_B=E_R+E_X+Q.$$

The symbol A denotes a constant and $|M_{fi}|^2$ is the matrix element connecting the initial and final states. $f(\mathbf{k})$ is the momentum distribution function for $\mathbf{k}$ lying in the momentum space $d\mathbf{k}$. $\rho(E_B)$ is the probability of finding the deuteron within the nucleus in the energy range dE_B.

In free p-d elastic scattering, we have

$$E_B=0 \quad \text{and} \quad f(\mathbf{k})=\delta(\mathbf{k}'). \tag{A2}$$

Then the cross section for elastic p-d scattering becomes

$$\frac{d\sigma}{d\mathbf{k}_p' d\mathbf{k}_d'} = A|M_{fi}|^2 \delta(E_0'-E_p'-E_d') \times\delta(\mathbf{k}_0'-\mathbf{k}_p'-\mathbf{k}_d'), \tag{A3}$$

where the primes refer to free p-d elastic scattering.

Integrating (A3) over $\mathbf{k}_d'$, we have

$$\frac{d\sigma}{d\mathbf{k}_p'} = A|M_{fi}|^2\delta(E_0'-E_p'-E_d'). \tag{A4}$$

Now we have

$$\frac{d\sigma}{d\mathbf{k}_p'} = \frac{\hbar^2}{mk_p'}\frac{d\sigma}{dE_p' d\Omega_p'}, \tag{A5}$$

where m is the proton mass. Integrating (A4) over E_p', we obtain

$$A|M_{fi}|^2 = \frac{\hbar^2}{mk_p'}\frac{d\sigma}{d\Omega_p'}. \tag{A6}$$

Substituting (A6) into (A1), we have

$$\frac{d\sigma}{d\mathbf{k}_p d\mathbf{k}_d} = \frac{\hbar^2}{mk_p'}\frac{d\sigma}{d\Omega_p'} f(\mathbf{k})\rho(E_B). \tag{A7}$$

Using

$$\mathbf{k}_p'+\tfrac{1}{2}\mathbf{k}=\mathbf{k}_p,$$
$$\mathbf{k}_d'+\mathbf{k}=\mathbf{k}_d,$$

we get

$$\frac{d\sigma}{d\Omega_p d\Omega_d dE_p dE_d} = \frac{2mk_p k_d}{\hbar^2 k_p'}\frac{d\sigma}{d\Omega_p'} f(\mathbf{k})\rho(E_B). \tag{A8}$$

APPENDIX B

As shown by Elliot and Lane[12] the completely antisymmetric wave function of the nucleons in the shell model with given L_0, S_0, and t_0 is

$$\psi_{L_0 M_0 t_0 t_{0Z} V_0} = \sum_{\bar{\psi}\phi_\alpha} (\psi[\![\bar{\psi})(\bar{\psi}[\![\phi_\alpha) \times\sum_{\phi_\alpha} W(L_\alpha \hat{L} L_0 \hat{L}; \bar{L}L) W(S_\alpha \hat{S} S_0 \hat{S}; \bar{S}S) \times W(t_\alpha \hat{t} t_0 \hat{t}; \bar{t}, t)\{\phi_\alpha(\boldsymbol{\tau}_{A-2}), \phi_\beta(1,2)\}_{L_0 S_0 t_0}, \tag{B1}$$

where V_0 are the quantum numbers characterizing the ground state of the nucleus, that is, the seniority and the reduced isospin. The symbols $(\alpha[\![\beta)$ are the fractional parentage coefficients and $\{\alpha,\beta\}_{L_0 S_0 t_0}$ defines the vector coupling of the states α and β to the L_0, S_0, t_0 state. The other symbols are given by Elliot and Lane.

Using the relation

$$\{\phi_\alpha,\phi_\beta\}_{L_0 S_0 t_0} = \sum (L_\alpha L M_\alpha M|L_0 M_0) \times(S_\alpha S\sigma_\alpha\sigma|S_0\sigma_0)(t_\alpha t t_{\alpha Z} t_Z|t_0 t_{0Z}) \times\phi_{L_\alpha M_\alpha S_\alpha \sigma_\alpha t_\alpha t_{\alpha Z}}\phi_{LM_L S\sigma t t_Z}, \tag{B2}$$

we have

$$C_{\alpha\beta} = \sum(\psi[\![\bar{\psi})(\bar{\psi}[\![\phi_\alpha)\sum W(L_\alpha \hat{L} L_0 \hat{L}; \bar{L}L) \times W(S_\alpha \hat{S} S_0 \hat{S}; \bar{S}S) W(t_\alpha \hat{t} t_0 \hat{t}; \bar{t}t) \times(L_\alpha L M_\alpha M_L|L_0 M_0)(S_\alpha S\sigma_\alpha\sigma|S_0\sigma_0) \times(t t_\alpha t_{\alpha Z} t_Z|t_0 t_{0Z}). \tag{B3}$$